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MARCH 1-7, 2000**

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**U. S. Department of Commerce**

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# Mathematical Model for Fire Phenomenon

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## Abstract

The constitutional and the verification methods for the mathematical modeling are briefly made the propositions. The introductory equations are formed the connection with zone model and field model. We had compared the results of calculation of the field model with the zone model.

## 1. Introduction

The mathematical models for the compartment fires in the buildings were various developed in the response of the purpose of utilizations. Figure 1 shows the modeling of the fire in the relationship from the real fire to the building designs for the fire safety engineering and/or the improvement of the fire regulations.

In this paper, we proposed the transformational equations from the results of calculations of field model to the zone model, and we inquired the adequacy for the zone model from the point of view in the field model.

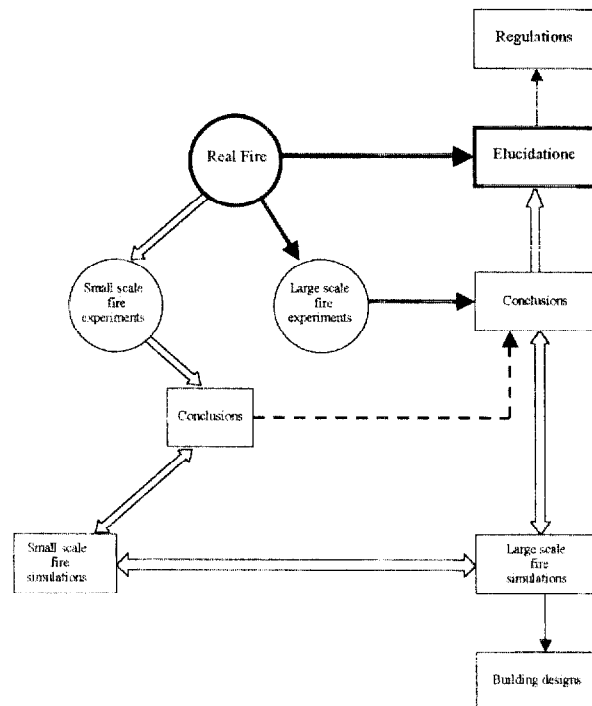


Figure 1. Fire and mathematical fire modeling.

## 2. Fire Modeling

When we shall make a mathematical modeling for the fire, we have to take the procedure into consideration in shown Fig 2. We have to make a “Physical Model” for the fire from abstraction of the “Real Fire Phenomenon” as the taking of experimental procedures over and over again. We can directly and qualitatively comprehend the fire phenomenon by the “Physical Model”. In order to have the quantitative results for the fire, we have to make the “Mathematical Model” which is abstracted and approximated the “Physical Model” again. The formulations of the most of “Mathematical Model” are introduced the nonlinear equations. We have hard the explicit solutions of these nonlinear formulae. The “Numerical Model” is needed to have the quantitative results of the “Mathematical Model” by using the numerical analysis and techniques and the computational performances. The “Simulation Model” is called another name of the “Numerical Model”. The “Simulation Code” is coded by the computer language and gives directly the quantitative results of the “Numerical Model”.

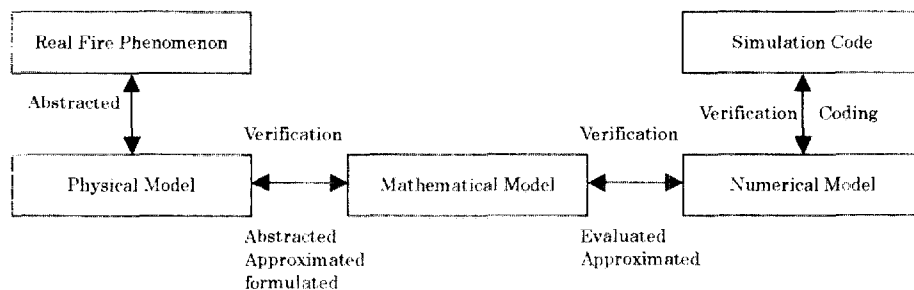


Figure 2. Modeling

When the calculational results from the simulation model are almost agreed with the measurements by the experiment, we are under the hallucination to be guaranteed the adequacy of the mathematical model. Unfortunately since we have simulated various case of fire by using this mathematical model, we have the erroneous results from this model. Moreover the results from this model are not a little applied to design and to evaluate the fire safety for the buildings, because of both the double abstractions and the double approximations from the real fire phenomenon to the mathematical model. Therefore we have to check the mathematical model from the physical model, and also physical model is checked the real fire phenomenon. It is not difficult to check the mathematical model to the numerical model by using the theoretical and numerical analysis of the nonlinear partial differential equations.

### 3. Fire simulation models

It is not difficult to convert the results of field model to the values of zone model on the average. The Navier-Stokes' type equations of the field model which are derived the law of mass, the momentum balance, and the energy balance, are indicated the time dependent nonlinear partial differential equation. On the other hand, the equations of the zone model which are derived the mass balance and energy balance without the momentum balance, are indicated the time dependent nonlinear ordinal differential equation by produced Harvard Code and Tanaka Code etc.

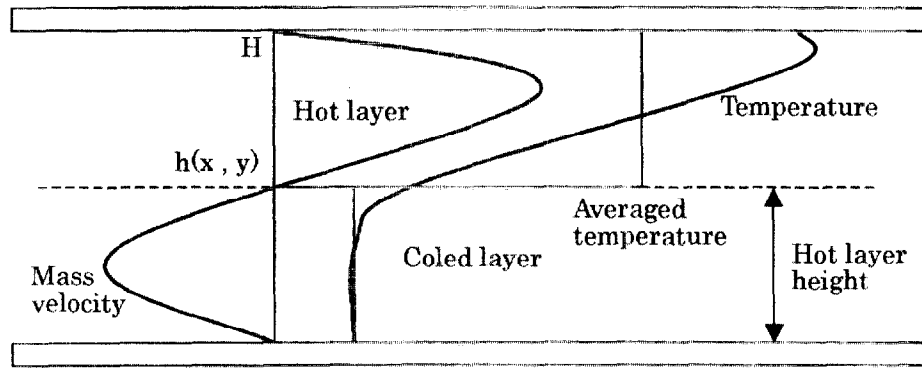


Figure 3. Outline of field and zone models

Following formula are the conversion forms with reference to Fig. 3.

(1) Temperature

$$\left\{ \begin{array}{l} \text{Hot layer} \quad \bar{\theta}_u = \frac{1}{L_x L_y} \int_0^{L_x} \int_0^{L_y} \frac{1}{H - h(x, y)} \int_{h(x, y)}^H \theta(x, y, z) dz dy dx \\ \text{Cold layer} \quad \bar{\theta}_l = \frac{1}{L_x L_y} \int_0^{L_x} \int_0^{L_y} \frac{1}{h(x, y)} \int_0^{h(x, y)} \theta(x, y, z) dz dy dx \end{array} \right.$$

(2) Layer height without plume zone.

$$\rho w(x, y, h(x, y)) = 0$$

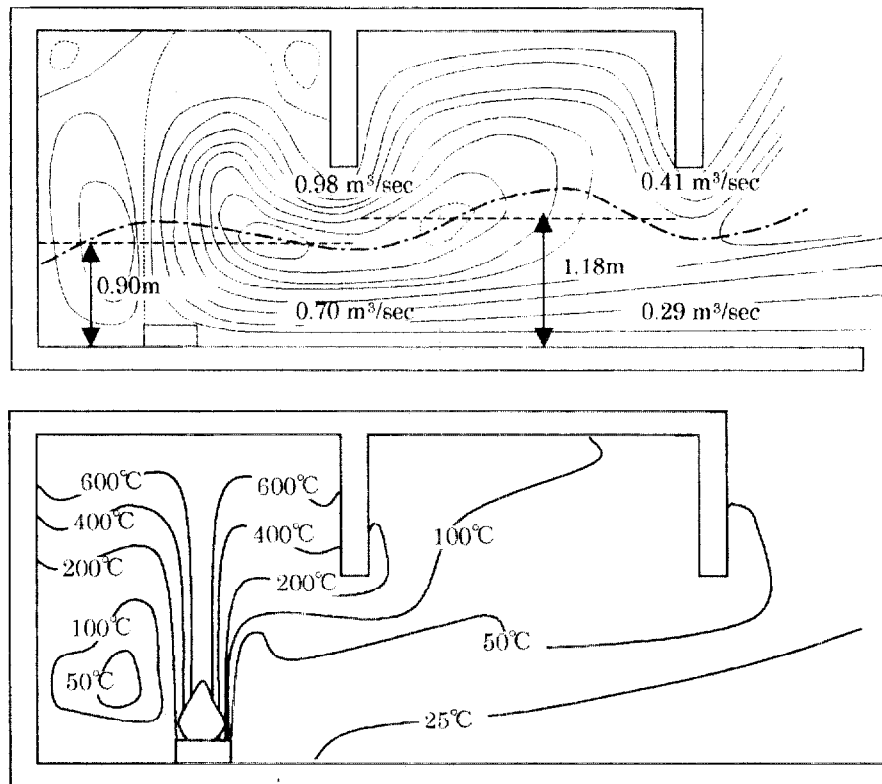
(3) Mass velocity at vents (x-direction at  $x_L$  location)

$$\left\{ \begin{array}{l} \text{Hot layer} \quad \overline{\rho w_u} = \frac{1}{L_y} \int_0^{L_y} \frac{1}{H - h(x_L, y)} \int_{h(x_L, y)}^H \rho w(x_L, y, z) dz dy \\ \text{Cold layer} \quad \overline{\rho w_l} = \frac{1}{L_y} \int_0^{L_y} \frac{1}{h(x_L, y)} \int_0^{h(x_L, y)} \rho w(x_L, y, z) dz dy \end{array} \right.$$

#### 4. Computational Results and Discussions

In this study, the field model and zone model are adopted for the mathematical models. We have calculated the Sutfire Simulation Code[1] for the field model and the Harvard VI Fire Simulation Code[2] for the zone model. The room sizes are both  $4.22 \times 3.35 \times 2.44$  m and the door sizes are both  $1.07 \times 2.03$  m.

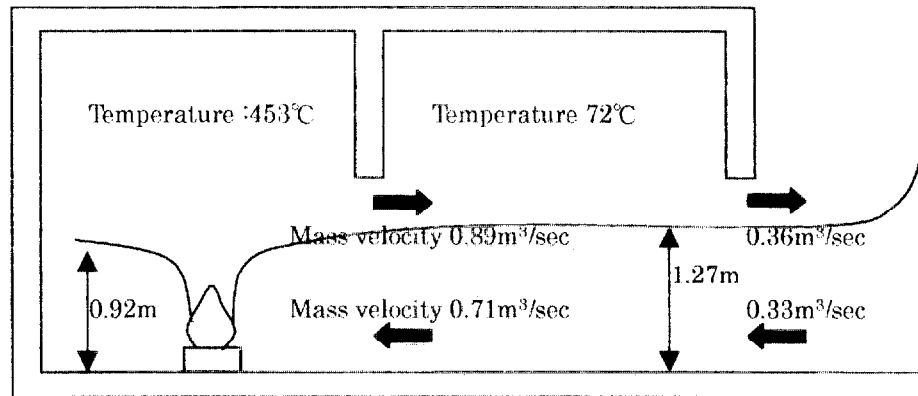
Figure 4 shows the calculational results of the distributions of the stream line and the temperature distributions by using Sutfire field model simulation code, and the mean values of the hot layer height and mass velocities at the vents.



*Figure 4. Calculation results for the field model*

The mean values of the neutral zone from the field model are about  $0.90 \text{ m}$  in the fire room and  $1.19 \text{ m}$  in neighbor room. The mass velocities at the vent are about  $0.98 \text{ m}^3/\text{sec}$  outflow and about  $0.71 \text{ m}^3/\text{sec}$  outflow at the door in the fire room, and about  $0.70 \text{ m}^3/\text{sec}$  outflow and about  $0.29 \text{ m}^3/\text{sec}$  outflow at the outside door in the neighbor room. The mean values of room temperature are about  $432^\circ\text{C}$  at the upper layer and about  $41^\circ\text{C}$  at the lower layer in the fire room.

Figure 5 shows the calculational results by using Harvard VI zone model fire simulation code



*Figure 5. Calculation results for the zone model*

We have a good agreement between the results of the zoon model and of the field model because of above figures.

## 5. References

- [1] Masahiro Morita et al, Feasibility of Numerical Computational Methods of Heat Flow in Fire Compartment, Fire Science and Technology, Vol.5, No.2, 159-164(1986)
- [2] John A. Rockett and Masahiro Morita, The NBS/Harvard Mark VI Multi-Room Fire Simulation, NBSIR 85-3281, 1986